Supersymmetric branes in the matrix model of a *pp* wave background

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We consider the matrix model associated with a *pp*-wave background and construct supersymmetric branes. In addition to the spherical membrane preserving 16 supersymmetries, one may construct rotating elliptic membranes preserving 8 supersymmetries. The other branch describes rotating 1/8 BPS hyperbolic branes in general. When the angular momentum vanishes, the hyperbolic brane becomes 1/4 Bogomol'nyi-Prasad-Sommerfield (BPS) preserving 8 real supersymmetries. It may have the shape of a hyperboloid of one or two sheets embedded in the flat three space. We study the spectrum of the worldvolume fields on the hyperbolic branes and show that there are no massless degrees. We also compute the spectrum of the 0-2 strings.

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I. INTRODUCTION

Recently there has been much attention to string theories in a *pp* wave background, which may be obtained from the Penrose limit of $AdS_p\times S^q$ geometries [1–21]. Remarkably the string spectrum in this background arises from a certain subsector of $N=4$ super-Yang-Mills theories [3], extending the usual AdS conformal field theory (CFT) dualities.

The matrix model of the *pp* wave background is also constructed in Ref. [3]. The related *pp* wave metric may be obtained from the Penrose limit of $AdS_7\times S^4$, which arises as a near horizon limit of M5 branes. Compared to the usual matrix model [22], the matrix model in a *pp* wave background includes mass terms and couplings to the background four form field strength. All the degrees become massive and the flat directions of the usual matrix model completely disappear, which makes the theory more accessible than the usual matrix model. Despite the presence of the mass term and couplings, the model in a *pp* wave background still possesses real 32 supersymmetries.

Because of the Myers effect [23] induced by the background four form field strength, the matrix theory allows 1/2 Bogomol'nyi-Prasad-Sommerfield (BPS) fuzzy sphere solutions [3]. The fuzzy membrane corresponds to the giant graviton wrapping two sphere of S^4 expanded by the effect of the four form flux [24]. Other simple time-dependent 1/4 BPS configurations are constructed there, which may be interpreted as a collection of D0 branes rotating around a fixed axis with an angular frequency corresponding to the natural frequency provided by the harmonic potentials of mass term. These are nothing to do with the expanded branelike configurations.

In this paper we study more general BPS brane solutions in the matrix model. For the related discussions on the D branes in a *pp* wave background, see Refs. [5,11,17,18,20,21]. The usual flat D2 branes cannot be supported in this model because all the coordinates acquire mass terms. Instead we find two branches of membrane solutions. The first describes rotating ellipsoidal branes preserving 8 real supersymmetries. By the effect of the rotation, the fuzzy sphere gets deformed to the ellipsoidal shape. Contrary to our expectation, the lengths of three axes of the ellipsoid become all different and $SO(3)$ invariance is broken down to Z_2. Within this branch, if one increases the angular momentum further, the shape of the brane becomes hyperbolic. The relevant BPS equations may be viewed as a deformation of the BPS equations arising in the matrix model description of the supersymmetric tubes [25–27].

The other branch in general describes a rotating hyperbolic membrane preserving 1/8 supersymmetries. The configuration is embedded in the three space formed by the two coordinates from AdS_7 and the remaining one along the S^4 direction. When the angular momentum vanishes in this branch, the configuration becomes 1/4 BPS. The brane takes the shape of hyperboloid. It could either take the shape of the upper/lower part of the two-sheet hyperboloid or the one-sheet hyperboloid (dS_2) embedded in the flat three space. We study the detailed solutions of the static hyperbolic branes. The relevant solutions are provided by the unitary representation of the $SO(2,1)$ algebra. There are five classes of unitary representation of $SO(2,1)$ and one may find the corresponding brane interpretation for all of the classes.

We further study the spectrum of the transverse scalar fields on the hyperbolic branes. The supersymmetric noncommutative solitons on the branes describe D0 branes located at the origin. The translational moduli are completely lifted by the mass terms. Using these solutions, we compute the spectrum of 0-2 strings connecting D0 branes to the hyperbolic branes.

In Sec. II we introduce the matrix model in a *pp* wave background and review the fuzzy membrane solutions and the collectively rotating D0 brane solutions. In Secs. III and IV, we study the supersymmetry conditions introducing relevant projection operators. In particular, we study the two classes of BPS states. We present the corresponding solutions, including the rotating ellipsoidal membrane, the static hyperbolic branes and the rotating hyperbolic branes. In Sec. V we discuss the detailed solutions of static hyperbolic branes using the unitary representation of SO(2,1) algebra and give the interpretation in terms of brane configurations. We study the spectrum of the scalar field on the worldvolume of hyperbolic brane. Using the noncommutative solitons on this brane, we investigate the spectrum of 0-2 strings.

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II. MATRIX THEORY AND SPHERICAL MEMBRANE

The matrix model in a *pp* wave is constructed in Ref. [3]. The related geometry is $AdS_7\times S^4$ and, in the Penrose limit, the metric and the four form fields become

$$ds^2=-4dx^-dx^+-\left(\frac{\mu}{6}\right)^2[4(x_1^2+x_2^2+x_3^2)+(x_4^2+\cdots+x_9^2)]\times(dx^+)^2+dx^idx^i,\quad F_{+123}=\mu, \tag{1}$$

where 1,2,3 directions were coordinates of S^4 and the remaining six directions are related to the spatial directions of the AdS_7. One of the angular directions in S^4 is used in the light cone coordinate. The matrix model Lagrangian [3] in this background becomes

$$L=L_0+L_\mu \tag{2}$$

with

$$L_0=\frac{1}{2R}\mathrm{tr}\left(\sum_i (D_0X_i)^2+R^2\sum_{i<j}[X_i,X_j]^2\right)+\mathrm{tr}\left(\psi^TD0\psi+i\sum_i R\psi^T\gamma_i[\psi,X_i]\right),$$

$$L_\mu=-\frac{1}{2R}\left(\frac{\mu}{6}\right)^2\mathrm{tr}\left(4\sum_{i=1}^3 X_i^2+\sum_{i=4}^9 X_i^2\right)-\frac{\mu}{4}\mathrm{tr}\,\psi^T\gamma_{123}\psi-\frac{\mu}{3}i\sum_{i,j,k=1}^3 \mathrm{tr}\,X_iX_jX_k\epsilon_{ijk}, \tag{3}$$

where we set the 11-dimensional Planck length $l_{11}=1$. $R=g_s\sqrt{\alpha'}$ is the radius of the direction $2x^-$ and l_{11} is related to the string scale by $l_{11}=(2\pi\alpha' R)^{1/3}$. The 16-dimensional Majonara spinors are used for the fermionic part and the gamma matrices are taken to be real. The scale R (together with l_{11}) will be omitted below by setting them at unity. The Lagrangian L_0 is the same as the usual matrix model in [22]. L_μ includes mass terms and the coupling to the four form field background.

The system possesses 32 real supersymmetries in total. The fields transform under the supersymmetry as

$$\delta X=2\psi^T\gamma_i\epsilon(t),\quad \delta A_0=2\psi^T\epsilon(t),$$

$$\delta\psi=\frac{1}{R}\left(D_0X_i\gamma_i+\frac{\mu}{3}\sum_{i=1}^3 X_i\gamma_i\gamma_{123}-\frac{\mu}{6}\times\sum_{i=4}^9 X_i\gamma_i\gamma_{123}+\frac{iR}{2}[X_i,X_j]\gamma_{ij}\right)\epsilon(t), \tag{4}$$

with

$$\epsilon(t)=e^{-(\mu/12)\gamma_{123}t}\epsilon_0. \tag{5}$$

For the remaining 16 supersymmetries, only fermions transform as $\delta\psi=e^{(\mu/4)\gamma_{123}t}\tilde{\epsilon}_0$ while all the bosonic coordinates do not change. Like the usual matrix model, this part of the supersymmetries are realized nonlinearly due to the presence of N D0 particles.

Because of the mass terms, the flat directions of the usual matrix model completely disappear. Furthermore, the coupling term to the four form field strength may induce the dielectric effect [23].

In this paper we shall focus on the nonperturbative states in the matrix model of a *pp* wave background. As found in Ref. [3], there are solutions preserving 1/2 supersymmetries. We briefly review this fuzzy sphere configuration for the later comparison. From the supersymmetric transformation of the fermionic coordinate, the BPS equation becomes

$$[X_i,X_j]=i\frac{\mu}{3}\epsilon_{ijk}X_k \tag{6}$$

for $i,j,k=1,2,3$, $D_0X_i=0$ for all i, and $X_4=\cdots=X_9=0$. The solutions are given by a fuzzy sphere; writing $X_i=(\mu/3)L_i$, one sees that

$$[L_i,L_j]=i\epsilon_{ijk}L_k \tag{7}$$

and solutions are given by a unitary representation of $SU(2)$. For the $n=2j+1$ dimensional representation with Casimir $j(j+1)$, the fuzzy sphere is described by

$$X_1^2+X_2^2+X_3^2=\left(\frac{\mu}{3}\right)^2 j(j+1)=\left(\frac{\mu}{6}\right)^2(n^2-1). \tag{8}$$

The translations do not give new solutions and the related moduli are lifted completely. Since 1,2,3 coordinates are related to S^4, it is clear that the above configuration wraps a sphere in S^4 and may be identified as a giant graviton in a *pp*-wave background [24].

In Ref. [3], solutions preserving 1/4 of supersymmetries were found. An example is

$$(X_4+iX_5)(t)=e^{\pm i(\mu/6)t}(X_4+iX_5)(0),\quad [X_4,X_5]=0. \tag{9}$$

In the basis diagonalizing X_4 and X_5 simultaneously, D0 branes taking definite time-dependent positions rotate around the origin at the same angular frequency as their natural frequency $\mu/6$ of the harmonic potential. Hence we see that the motion of D0 branes are the usual particle motion in the harmonic potential, which is nothing to do with the formation of expanded higher-dimensional branes.

Of course, similar solutions may be obtained by replacing Eqs. (4), (5) with any other pairs. In addition, there is a similar 1/4 BPS solution by taking a pair of indices from 1,2,3 with time dependence $e^{\pm i(\mu/3)t}$. Again the angular frequency and the natural frequency agree to each other.

III. ROTATING ELLIPSOIDAL BRANES

In this section we shall investigate more closely the case where only the first three components of X_i are turned on. The resulting configuration in general preserves 1/4 supersymmetries restoring 1/2 supersymmetry in the static limit.

As we will see, the configuration corresponds to a deformation of the spherical membrane to a time-dependent ellipsoidal one. There is at least one parameter family of solutions describing the deformation. If this parameter approaches a certain critical value, the ellipsoid collapses in two directions. Beyond the critical value, the membrane surface takes a shape of hyperboloid.

From the combination of $\gamma_i (i=1,2,3)$, the only real projection operator is of the type

$$P^1_{\pm}=\frac{1\pm\hat{n}_i\gamma_i}{2}, \tag{10}$$

where $\hat{n}_i$ is a real vector of unit size. Without loss of generality, we shall choose $\hat{n}_i$ to the 3 direction using the global $SO(3)$ symmetry.

The supersymmetry condition, $\delta\psi=0$, then leads to BPS equations,

$$i[X_1,X_2]+\frac{\mu}{3}X_3=0,\ \ D_0X_3=0,$$

$$i[X_1,X_3](\pm)-\frac{\mu}{3}X_2(\pm)+D_0X_1=0,$$

$$i[X_2,X_3](\pm)+\frac{\mu}{3}X_1(\pm)+D_0X_2=0, \tag{11}$$

by setting the coefficient of $P^1_{\pm}\epsilon_0$. For the remaining supersymmetries, we shall satisfy the equations for just one choice of the sign. In addition, there is the Gauss law constraint,

$$[X_1,D_0X_1]+[X_2,D_0X_2]=0. \tag{12}$$

For definiteness, we shall choose the "+" sign projection corresponding to the remaining supersymmetries satisfying $\epsilon_0=-\gamma_3\epsilon_0$. Working in a gauge $A_0=X_3$, the last two equations reduce to

$$\dot{X}_1+i\dot{X}_2=-\frac{\mu}{3}i(X_1+iX_2), \tag{13}$$

whose solution reads

$$X_1+iX_2=e^{-(\mu/3)it}(X_0+iY_0) \tag{14}$$

with constant Hermitian matrices X_0 and Y_0. The second equation in Eqs. (11) implies that $X_3\equiv Z$ is constant in time. Using this solution, the full set of BPS equations are reduced to

$$[X_0,[X_0,Z]]+[Y_0,[Y_0,Z]]-2\left(\frac{\mu}{3}\right)^2Z=0,$$

$$[X_0,Y_0]=i\frac{\mu}{3}Z. \tag{15}$$

These generalize the BPS equations associated with the supersymmetric tubular branes by a mass parameter [26,27]. The general solutions of these coupled equations are not known.

When $Z=0$, the above equations may be trivially satisfied and the solutions in Eq. (14) with commuting X_0 and Y_0 describe the rotating 1/4 BPS D0 branes discussed in the preceding section.

For more general solutions, we try the following ansatz:

$$[Z,X_0]=i\frac{\mu}{3}(1+a)Y_0,\quad [Y_0,Z]=i\frac{\mu}{3}(1-b)X_0. \tag{16}$$

The first equation in Eqs. (15) then implies that $a=b$. The momentum becomes

$$D_0X_1+iD_0X_2=-i\frac{\mu a}{3}e^{-(\mu/3)it}(X_0-iY_0). \tag{17}$$

When $|a|\leqslant 1$, the solution may be presented as

$$X_0=\frac{\mu}{3}\sqrt{1+a}L_1,\ \ Y_0=\frac{\mu}{3}\sqrt{1-a}L_2,\ \ Z=\frac{\mu}{3}\sqrt{1-a^2}L_3 \tag{18}$$

with the generators of the SO(3) algebra $[L_i,L_j]=i\epsilon_{ijk}L_k$. The shape is determined by the Casimir of the SO(3) representation and given by

$$\frac{X_0^2}{1+a}+\frac{Y_0^2}{1-a}+\frac{X_3^2}{1-a^2}=\left(\frac{\mu}{3}\right)^2j(j+1). \tag{19}$$

When $a=0$, the solution becomes spherical and $D_0X_1=D_0X_2=0$. Though we are using here a different gauge, this corresponds to the 1/2 BPS spherical membrane described in the previous section. Otherwise the solutions describe rotating ellipsoidal branes, which is 1/4 BPS. The total energy for this configuration and the angular momentum are evaluated as

$$H=\left(\frac{\mu a}{3}\right)^2\mathrm{tr}(X_0^2+Y_0^2)=\frac{a^2}{6}\left(\frac{\mu}{3}\right)^4n(n^2-1),$$

$$J_{12}=\mathrm{tr}(X_1D_0X_2-X_2D_0X_1)=-\frac{a^2}{6}\left(\frac{\mu}{3}\right)^3n(n^2-1) \tag{20}$$

for the n-dimensional representation of the SO(3) algebra. As we see from Eq. (19), the rotation in the 12 plane makes the shape of the brane elliptical. However, the deformation is more than expected. The lengths of three axes become all different and SO(3) R symmetry of the model is broken down to at most discrete subgroups.

For $|a|>1$, the rotating membrane takes the shape of a hyperboloid. Specifically for $a>1$, the solution may be presented as

$$X_0 = \frac{\mu}{3}\sqrt{1+a}K_1, \quad Y_0 = \frac{\mu}{3}\sqrt{a-1}K_2, \quad Z = \frac{\mu}{3}\sqrt{a^2-1}K_3 \tag{21}$$

with the generators of the SO(2,1) algebra $[K_1,K_2]=iK_3$, $[K_3,K_1]=iK_2$ and $[K_2,K_3]=-iK_1$. The Casimir operator in this case is $K=K_2^2+K_3^2-K_1^2$ and the shape is described by

$$\frac{X_0^2}{1+a} - \frac{Y_0^2}{a-1} - \frac{X_3^2}{a^2-1} = -\left(\frac{\mu}{3}\right)^2 K. \tag{22}$$

There is a nonvanishing angular momentum in the 12 plane. The detailed representation of the SO(2,1) algebra will be presented in the next section where we discuss the static hyperboloid.

IV. ROTATING 1/8 BPS HYPERBOLIC BRANES

To look for the other BPS configurations, let us turn on one component out of 1,2,3 directions and two components out of the remaining six directions. Specifically we turn on X_3, X_4 and X_5. The unbroken supersymmetry condition, $\delta\psi=0$, may be written as

$$M_+\left(D_0X_4\gamma_4 + D_0X_5\gamma_5 - \frac{\mu}{6}(X_4\gamma_4+X_5\gamma_5)\gamma_{123} + i[X_4,X_3]\gamma_{43} + i[X_5,X_3]\gamma_{53}\right)\epsilon_0 + M_-\left(D_0X_3\gamma_3 + \frac{\mu}{3}X_3\gamma_3\gamma_{123} + i[X_4,X_5]\gamma_{45}\right)\epsilon_0 = 0, \tag{23}$$

where $M_\pm \equiv e^{\pm(\mu/12)\gamma_{123}t}$. We now introduce real projection operators

$$P_\pm^1 = \frac{1\pm\gamma_3}{2}, \quad P_\pm^5 = \frac{1\pm\gamma_{12345}}{2} \tag{24}$$

and it is straightforward to see that the above equation reduces to

$$i[X_4,X_5](+)(\pm) = \frac{\mu}{3}X_3(\pm)(+), \qquad D_0X_3=0,$$

$$i[X_4,X_3](\pm)(+) = \frac{\mu}{6}X_5(+)(\pm) - D_0X_4,$$

$$i[X_5,X_3](\pm)(+) = -\frac{\mu}{6}X_4(+)(\pm) - D_0X_5 \tag{25}$$

by setting coefficients of $P_\pm^1 P_\pm^5 \epsilon_0$ to zero. Here the signatures in the first parentheses of each term represent that of the projection operator $P_\pm^1$ whereas the second signatures for $P_\pm^5$ $(+)$ represents that the projection operators are not involved. The configuration should satisfy the Gauss law,

$$[X_4,D_0X_4]+[X_5,D_0X_5]=0. \tag{26}$$

For the remaining supersymmetries, the above equations should satisfy at least one set of the choice of the signatures. Let us choose the $P_+^1P_-^5$ case. The other choices can be treated similarly. When $Z=0$, $[X_4,X_5]=0$ follows from the first equation of Eq. (25). The remaining equation reduces to the 1/4 BPS equation (9) discussed in the preceding section with the remaining supersymmetries given by $P_\pm^5\epsilon_0$.

Nonrotating solutions are given only if $D_0X_4=D_0X_5=0$. The corresponding BPS equations become

$$[X_4,X_5] = +i\frac{\mu}{3}X_3, \quad [X_5,X_3] = -i\frac{\mu}{6}X_4,$$

$$[X_3,X_4] = -i\frac{\mu}{6}X_5. \tag{27}$$

The configuration is 1/4 BPS and the remaining supersymmetries are given by the projection $(P_+^1P_-^5 + P_-^1P_+^5)\epsilon_0$. This projection is equivalent to a single projection operator P_-^4 defined by

$$P_\pm^4 \equiv \frac{1\pm\gamma_{1245}}{2}. \tag{28}$$

One could use $P_+^1P_+^5 + P_-^1P_-^5 = P_+^4$ by which again we get SO(2,1) algebra corresponding to the parity operation of Eq. (27). Later we shall analyze details of this static case.

For a more general case, again we choose a gauge $A_0=X_3$ and the projection $P_+^1P_-^5$. The last two BPS equations then become

$$\dot{X}_4 + i\dot{X}_5 = \frac{\mu}{6}i(X_4+iX_5), \tag{29}$$

whose solution reads

$$X_4 + iX_5 = e^{(\mu/6)it}(X_0+iY_0) \tag{30}$$

with constant Hermitian matrices X_0 and Y_0. The second equation in Eq. (11) implies that $X_3\equiv Z$ is constant in time. Using this solution, the BPS equations are reduced to

$$[X_0,[X_0,Z]] + [Y_0,[Y_0,Z]] + \left(\frac{\mu}{3}\right)^2 Z = 0,$$

$$[X_0,Y_0] = i\frac{\mu}{3}Z. \tag{31}$$

The essential difference from Eq. (15) is the signature in front of the mass square term. These equations in general describe 1/8 BPS configurations preserving four real supersymmetries. Again we do not know how to solve the equation in general. Taking the ansatz

$$[Z,X_0] = -i\frac{\mu}{6}(1+a)Y_0, \quad [Y_0,Z] = -i\frac{\mu}{6}(1-b)X_0, \tag{32}$$

one finds $a=b$ from Eq. (31). The momentum in 45 directions then becomes

$$D_0X_4+iD_0X_5=i\frac{\mu a}{6}(X_0-iY_0)e^{(\mu/6)it}. \tag{33}$$

The $a=0$ case corresponds to the 1/4 BPS configuration with vanishing momentum. For general a, the configuration takes a shape

$$\frac{X_0^2}{2(1+a)}+\frac{Y_0^2}{2(1-a)}-\frac{X_3^2}{1-a^2}=\left(\frac{\mu}{6}\right)^2\text{Casimir}, \tag{34}$$

rotating in the 45 plane.

The total energy and the angular momentum for this 1/8 BPS configuration are evaluated as

$$H=\left(\frac{\mu}{6}\right)^2\text{tr}[(a^2-a+1)X_0^2+(a^2+a+1)Y_0^2+4Z^2],$$

$$J_{45}=\text{tr}(X_4D_0X_5-X_5D_0X_4)=\frac{a\mu}{6}\text{tr}(X_0^2-Y_0^2). \tag{35}$$

As $|a|$ grows, the angular momentum in general increases and the shape changes accordingly. But in this 1/8 BPS branch, the instant shape of the brane configuration always takes the form of the hyperboloid as we see in Eq. (34).

V. STATIC HYPERBOLIC BRANES AND THEIR FLUCTUATION SPECTRUM

In the preceding section we have obtained static 1/4 BPS configurations. With definitions

$$X_4=\frac{\mu}{3\sqrt{2}}K_1, \quad X_5=\frac{\mu}{3\sqrt{2}}K_2, \quad Z=\frac{\mu}{6}K_3, \tag{36}$$

the BPS equations in Eqs. (27) implies that K_i satisfies the SO(2,1) algebra. Namely, $[K_1,K_2]=iK_3$, $[K_3,K_1]=-iK_2$ and $[K_2,K_3]=-iK_1$, which we write as $[K_i,K_j]=if_{ij}^kK_k$ using a SO(2,1) structure constant f_{ij}^k. The Casimir operator in this case is $K=K_1^2+K_2^2-K_3^2$ and the shape of the configuration is described by

$$\frac{1}{2}(X_4^2+X_5^2)-X_3^2=\left(\frac{\mu}{6}\right)^2K. \tag{37}$$

The BPS branes are classified by all possible unitary representations of SO(2,1) algebra, which have been already worked out in the literature [28–30]. The basic idea is to introduce the step operators by

$$K_\pm=K_1\mp iK_2, \tag{38}$$

which satisfy the commutation relations

$$[K_3,K_\pm]=\pm K_\pm, \quad [K_-,K_+]=2K_3. \tag{39}$$

Let us work in a basis diagonalizing K and K_3, the eigenvalues of which are denoted, respectively, by k and m. Using $K_-=K_+^\dagger$ in the unitary representation, the relation

$$K_\mp K_\pm=K+K_3(K_3\pm1) \tag{40}$$

gives a requirement

$$k+m(m\pm1)\geqslant0, \tag{41}$$

for all m. Then there are basically the following five class of unitary representations parametrized by j with $k=-j(j-1)$.

(1) *Discrete representations* $\mathcal{D}_j^+$. The representation is realized in the Hilbert space

$$\mathcal{D}_j^+=\{|jm\rangle;m=j,j+1,j+2,j+3\cdots\}, \tag{42}$$

where $|jj\rangle$ is annihilated by K_-. j is real and positive for the unitary representation.

(2) *Discrete representations* $\mathcal{D}_j^-$. The Hilbert space is

$$\mathcal{D}_j^-=\{|jm\rangle;m=-j,-j-1,-j-2,\cdots\}, \tag{43}$$

where $|j-j\rangle$ is annihilated by K_+. j is real and positive for the unitary representation.

(3) *Continuous representations* $\mathcal{C}_j^\alpha$. The Hilbert space is

$$\mathcal{C}_j^\alpha=\{|\alpha;jm\rangle;m=\alpha,\alpha\pm1,\alpha\pm2,\cdots\}, \tag{44}$$

and $0\leqslant\alpha<1$ without loss of generality. The representation is unitary if $j=1/2+is$ with real s.

(4) *Complementary continuous representations* $\mathcal{E}_j^\alpha$. The Hilbert space is

$$\mathcal{E}_j^\alpha=\{|\alpha;jm\rangle;m=\alpha,\alpha\pm1,\alpha\pm2,\cdots\}, \tag{45}$$

and $0\leqslant\alpha<1$ without loss of generality. For $1/2<j<1$ with $j-1/2<|\alpha-1/2|$, the representation becomes unitary.

(5) *Identity representation*. This is a trivial representation with $j=m=0$.

All of the nontrivial unitary representations are solutions describing 1/4 BPS configurations. The case $\mathcal{D}_j^+$ describes a hyperbolic plane corresponding to the upper plane of the two-sheet hyperboloid as depicted in Fig. 1. There is a rotational symmetry in the 45 plane. In this case, the Casimir operator takes a value $k=1/4-(j-1/2)^2$ with $j>0$. For $j\geqslant1$, the upper part of the plane defined by Eq. (37) precisely agrees to the one in Fig. 1. When $0<j<1$, the plane defined by Eq. (37) takes the shape of dS_2. Because the eigenvalue of X_3 is bounded below in this case, we view that the brane still takes the shape of the upper plane of the two-sheet hyperboloid in the figure even for $0<j\leqslant1$. Presumably the effect of noncommutativity on the brane induces this kind of correction and makes the tip closed off.

One may understand the configuration as follows. Without a *pp* wave background, there is a usual M2 brane in the 45 plane that shares one direction with M5 branes. Now consider the situation where one holds the circular boundary of a large M2 brane at a fixed X_3 location. The mass term in the presence of a *pp* wave corresponds harmonic confining potentials in the target space. Since the region at the origin has a lower potential, the M2 brane gets deformed toward the origin in order to take advantage of the lower potential. Of course, there is a cost of energy by the increase of area

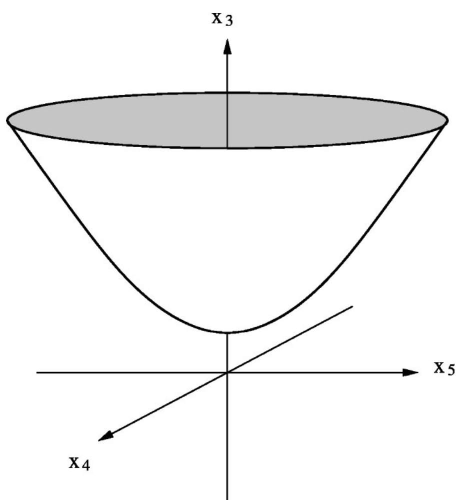

FIG. 1. We illustrate here the upper plane of a two-sheet hyperboloid.

together with a curving of the brane. These contributions are balanced in the above configurations.

The other discrete representation $\mathcal{D}_j^-$ corresponds to the lower part of a two-sheet hyperboloid. From $\mathcal{D}_j^+$, one may obtain $\mathcal{D}_j^-$ by the transformation $X_3 \to -X_3$.

The continuous representation corresponds to a brane of the shape dS_2. Namely the upper and lower parts of the two-sheet hyperboloid are connected by a tubular throat. The tube, in general, does not close off along the throat. Also the D0 branes are located along the X_3 direction with equal spacing $\Delta = \mu/6$, which one can read off from the eigenvalues of X_3. For $\mathcal{C}_j^\alpha$, $k = 1/4 + s^2$, which determines the classical shape. Especially, the fatness of the throat is determined by the Casimir. Unlike the case of the discrete representations, there is an extra parameter α for a given k. Since $D0$ branes in the X_3 directions are located with the equal spacing Δ, the configuration has a periodicity Δ along X_3 directions about the locations of D0's. Hence the parameter $\mu\alpha/6$ $(0 \leqslant \alpha < 1)$ precisely describes the locations of a D0 brane within $0 \leqslant x_3 < \Delta$.

For $\mathcal{E}_j^\alpha$, $k = 1/4 - (j-1/2)^2 \in (0,1/4)$ with $1/2 < j < 1$ and the throat becomes narrower as j approaches 1. The effect of the noncommutativity along the throat becomes crucial in this case. In particular, the freedom of locating a $D0$ brane in the unit interval gets lifted due to the too narrow throat. And the lattice translational freedom of the $D0$ brane location breaks down partially. By this one may understand the restriction of the locational parameter α given by $j - 1/2 < |\alpha - 1/2|$. The detailed dynamical investigation of the k dependence of the D0 brane location will be quite interesting.

Finally we like to discuss the spectrum of the fluctuation around the hyperbolic brane background. For simplicity, we like to take the case of the upper part of a two-sheet hyperboloid of the representation $\mathcal{D}_j^+$. Among the degrees, we like to concentrate on the transverse fluctuation along 6,7,8,9. Let us call them ϕ. Then the effective Lagrangian governing the quadratic fluctuation becomes

$$L_{\rm eff} = \frac{1}{2}\mathrm{tr}\left[\dot{\phi}^2 - \left(\frac{\mu}{6}\right)^2 \phi^2 - \sum_{i=3,4,5} \phi[\overline{X}_i,[\overline{X}_i,\phi]]\right], \quad (46)$$

where $\overline{X}_i$ denotes the brane solution. Here we choose a gauge $A_0 = 0$. In order to find the spectrum, one has to diagonalize the above effective action, for which we proceed as follows. The scalar field in general may be written in components as

$$\phi = \sum_{nm} \phi_{nm} |jn\rangle\langle jm|. \quad (47)$$

We now define

$$\tilde{K}_i \phi \equiv -\phi K_i, \quad (48)$$

which acts on $\langle jm|$. Then it is simple to show that they satisfy the SO(2,1) algebra, i.e. $[\tilde{K}_i, \tilde{K}_j] = i f_{ij}^k \tilde{K}_k$. Hence,

$$[K_i,\phi] = (K_i + \tilde{K}_i)\phi = J_i \phi \quad (49)$$

with the total generator $J_i = K_i + \tilde{K}_i$. Also note that the bases $\langle jm|$ form the representation $\mathcal{D}_j^-$ with respect to $\tilde{K}_i$. The problem is then reduced to the decomposition of the product representation of $\mathcal{D}_j^+ \otimes \mathcal{D}_j^-$ into the sum of irreducible representation of the $SO(2,1)$ algebra. They are given by [30]

$$\mathcal{D}_j^+ \otimes \mathcal{D}_j^- = \int_{s=0}^{\infty} \mathcal{C}_{1/2+is}^{\alpha=0}. \quad (50)$$

Hence in these bases denoted by $\phi_{(s,n)}$ with non-negative s and integer n, one has

$$\sum_{i=3,4,5} [\overline{X}_i,[\overline{X}_i,\phi_{(s,n)}]] = \left(\frac{\mu}{6}\right)^2 \left(\frac{1}{2} + 2s^2 + 3n^2\right) \phi_{(s,n)}. \quad (51)$$

By adding the contribution from the original mass term in Eq. (46) which is already diagonal in the basis $\phi_{(s,n)}$, the total mass squared is

$$M_{s,n}^2 = \left(\frac{\mu}{6}\right)^2 \left(\frac{3}{2} + 2s^2 + 3n^2\right). \quad (52)$$

Similarly one may compute the mass spectrum of the scalar in (1,2) directions and they are

$$\tilde{M}_{s,n}^2 = \left(\frac{\mu}{6}\right)^2 \left(\frac{9}{2} + 2s^2 + 3n^2\right). \quad (53)$$

From the expressions, it is clear that there are no massless modes and there are no moduli related to the brane position in the transverse space.

Finally we like to mention the spectrum of 0-2 strings connecting the upper brane of a two-sheet hyperboloid and D0 branes sitting at the origin. The D0 branes may be given as a localized noncommutative soliton solution [31–34] from the view point of the worldvolume theory of the brane. For simplicity, we consider the representation $\mathcal{D}_j^+$. It is straightforward to check that the following solution satisfies the BPS algebra (27):

$$X_4=\frac{\mu}{3\sqrt{2}}SK_1S^\dagger,\quad X_5=\frac{\mu}{3\sqrt{2}}SK_2S^\dagger,\quad Z=\frac{\mu}{6}SK_3S^\dagger, \tag{54}$$

where we define a shift operator S such that

$$S^\dagger S=I,\quad SS^\dagger=I-P \tag{55}$$

with an l-dimensional projection operator P. The solution describes l D0 branes sitting at the origin [32,34]. Again due to the mass terms in the model, the translational moduli of D0 branes completely disappear. In this BPS configuration, they are located at the origin, which is the minimum point of the potential. More explicitly, for the projection operator $P=|jj\rangle\langle jj|$, the corresponding shift operator may be constructed as

$$S=\sum_{n=0}^{\infty}|j,j+n+1\rangle\langle j,j+n|. \tag{56}$$

In the background of this solution, one may compute the spectrum related to the 0-2 string. In particular ϕ in the transverse directions decomposed as $\phi=P\phi P+P\phi\bar{P}+\bar{P}\phi P+\bar{P}\phi\bar{P}$ with $\bar{P}=I-P$. The quadratic part of each term contributes to the action independently. One may then explicitly check that $P\phi P$ describes the fluctuation of D0's while the $\bar{P}\phi\bar{P}$ part describes the fluctuation of the original hyperbolic brane. The remaining part has to do with the 0-2 strings. Namely, they describe the interaction between the D0 branes and the hyperbolic brane. In the transverse direction, the spectrum may be evaluated straightforwardly. (See Refs. [32,34] for the detailed methods.) The resulting spectrum for $P\phi\bar{P}$ in the 6,7,8,9 directions reads

$$M_{na}^2=\left(\frac{\mu}{6}\right)^2[1+2j(1-j)+3(j+n)^2], \tag{57}$$

where a labels the ath D particle and $n=0,1,2,\ldots$. For the 1,2 direction, the mass spectrum is given by

$$\tilde{M}_{na}^2=\left(\frac{\mu}{6}\right)^2[4+2j(1-j)+3(j+n)^2]. \tag{58}$$

In these expressions, the constant parts, $1\times(\mu/6)^2$ or $4\times(\mu/6)^2$, come from the original mass terms of the Lagrangian (3). The remaining parts take the same form and may be interpreted as the distance squared between D0's at the origin and the nth D0 brane on the hyperbolic brane. More explicitly, the distance may be measured as $d_n^2=\langle jj+n|(X_4^2+X_5^2+X_3^2)|jj+n\rangle$ on the solution, which agrees with the remaining parts of the masses squared.

VI. CONCLUSION

In this paper we study the generic two brane solutions arising in the matrix model of a *pp* wave background. There are two branches we have identified. One is the 1/4 BPS branch where the rotating branes take an ellipsoidal shape. The others include 1/4 BPS static hyperbolic branes and rotating hyperbolic branes preserving 1/8 of the supersymmetries. We focus on the static branes of the hyperbolic shape and find all possible solutions classified by the unitary representation of the SO(2,1) algebra. We have computed the spectrum of the transverse fluctuation as well as 0-2 strings.

In the matrix model of a *pp* wave background, it is clear that the static flat branes are not supported because of mass terms. Also, the supersymmetric charges in this system are time dependent and do not commute with the Hamiltonian, which makes it difficult to apply the conventional methods of finding BPS equations. In this respect, we do not know the whole classifications of possible BPS equations in the system. Also for the BPS equations identified in this paper, the general solutions are not known and our solutions are obtained by taking specific ansatz. A more systematic understanding of the BPS states in this massive matrix model is necessary.

The detailed dynamical understanding of the formation or deformation of the branes are still lacking. For the rotating ellipsoidal shape of branes, the lengths of all the axes becomes different. If one increases the angular velocity, the ellipsoidal brane opens up and becomes hyperbolic. Why these are so dynamic is not clear to us. Further investigation is necessary.

We have computed the spectrums of the 0-2 string connecting D0 to the static hyperbolic brane. This may be directly compared with those of string theory in a *pp* wave background.

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